Constraining cosmic superstrings with dilaton emission

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Abstract

Brane inflation predicts the production of cosmic superstrings with tension $10^{-12} \lesssim G\mu \lesssim 10^{-7}$. Superstring theory predicts also the existence of a dilaton with a mass that is at most of the order of the gravitino mass. We show that the emission of dilatons imposes severe constraints on the allowed evolution of a cosmic superstring network. In particular, the detection of gravitational wave burst from cosmic superstrings by LIGO is only possible if the typical length of string loops is much smaller than usually assumed.

PACS: 04.30.Db, 11.27.+d, 98.80.Cq Preprint MPP-2005-11

1 Introduction

The inflationary paradigm has become one of the corner stones of modern cosmology. While this paradigm solves many puzzles of the "old" big bang theory, no convincing theoretical framework exists at present for the inflaton field and its potential. In the brane inflation scenario [1], the inflaton is identified as a mode controlling the separation of two branes. Inflation ends when the branes collide and heat-up the Universe. An intriguing prediction of most scenarios of brane inflation is the copiously production of cosmic superstrings [2].

One of the few generic predictions of superstring theory is the existence of the dilaton supermultiplet in its spectrum. Independent of the particular mechanism of supersymmetry breaking, any realistic string theory model should led to a low-energy theory with softly broken supersymmetry at the TeV scale in order to solve the hierarchy problem. This ensures together with other phenomenological constraints that the dilaton mass is at most of the order of the gravitino mass $m_{3/2}$ [3]. In hidden sector models of supersymmetry breaking

$m_{3/2}$ is around the electroweak scale, while in gauge-mediated supersymmetry breaking models $m_{3/2}$ could be as low as 0.01 keV.

Cosmological consequences of dilatons in string theories have been already widely discussed [3,4,5]. The purpose of this work is the investigation of dilaton emission from a network of cosmic superstrings. We derive stringent limits on the tension of the strings as function of the dilation mass and discuss the maximal gravitational wave signal from cosmic strings compatible with these limits.

2 Dilaton radiation from a cosmic superstring network

In this section, we reanalyze the emission of dilatons by a cosmic string network. Our main aim is the generalization of the analysis of Damour and Vilenkin in Ref. [5] to the case of a more general network evolution, where the energy losses of strings as well as their reconnection probability can deviate from their "standard" values.

Let us first recall the main quantities describing the standard evolution of a string network. The network is characterized by the typical length scale $L^{\rm st}(t) \sim t/\sqrt{\zeta}$, where $\zeta \sim 1/4$ is a parameter that has to be determined in numerical simulations. Another characteristic quantity of the cosmic string network is the parameter β characterizing the length l of loops which are chopped-off long strings,

$$l(t) \sim \beta t \,. \tag{1}$$

In the standard scenario, this coefficient β is determined by the gravitational radiation losses from loops,

$$\beta^{\rm st} \sim \Gamma G\mu \,. \tag{2}$$

Here, Γ is a numerical coefficient of the order 50. At any given moment of time t, the typical size of loops is given by inserting into Eq. (1) the appropriate value of β. Then the density of such loops $n^{\rm st}$ can be written as

$$n^{\rm st} \sim \frac{\zeta}{\Gamma G\mu\, t^3} \,. \tag{3}$$

The evolution of a string network can be modified in two ways: First, the probability of intercommuting of intersecting strings may be different from one, $p \leq 1$, as it happens for cosmic superstrings [6]. Second, the value of the

coefficient β can differ from the one usually assumed, $\beta \neq \Gamma G\mu$. Note that the first modification of the properties of cosmic strings ($p \neq 1$) is due to the difference between superstrings and topological strings, while the possible deviation of β from the standard value applies for both type of strings. Introducing a non-standard value for β is inspired by Ref. [9] where it was argued that β could be much smaller than it was originally assumed. It is convenient to introduce the ratio

$$\epsilon = \frac{\beta}{\beta^{\rm st}}. \tag{4}$$

Following [8], one can find the resulting differences in the properties of strings for the two extreme cases $\epsilon \ll 1$ and $\epsilon \gg 1$. In the latter case, the typical length of loops is given by

$$l_1 \sim \Gamma G\mu\, t, \tag{5}$$

and their density is

$$n_1 \sim \frac{\epsilon^{1/2}\zeta}{p\,\Gamma G\mu\, t^3}\,. \tag{6}$$

(Note that we are interested in the radiation domination epoch, while in [8] the matter domination epoch was considered. This leads to different expressions for the loop density.) In the opposite case, $\epsilon \ll 1$, one obtains as typical length of loops

$$l_2 \sim \beta\, t\,, \tag{7}$$

and as density of loops

$$n_2 \sim \frac{\zeta}{p\,\Gamma G\mu\, t^3}\,. \tag{8}$$

Let us now turn to the calculation of the number of dilatons radiated from a cosmic superstring network. In Ref. [5], it was shown that a single loop radiates dilatons of frequency ø with the rate

$$\dot{N}_\phi = \Gamma_\phi\, \alpha^2\, G\mu^2/ø. \tag{9}$$

The coefficient Γ_ϕ does not depend on the loop size and is typically $\Gamma_\phi \sim 13$, while $\alpha = \partial \ln\sqrt{\mu(\phi)}/\partial\phi$ measures the strength of the coupling of the dilaton to the string relative to the gravitational strength. To simplify the analysis,

we assume that all dilatons are emitted at the same fundamental frequency $ø = 4\pi/l$. Then the total number of radiated dilatons can be written as $N_\phi \sim \dot{N}_\phi \tau$, where τ is the decay time of a loop. For $\epsilon \gg 1$, this time is simply $\tau \sim t$. Since the typical length of loops is for $\epsilon \ll 1$ smaller by a factor ϵ, the decay time of a typical loop is by the same factor suppressed, $\tau \sim \epsilon t$. As result we obtain from Eq. (9) as total number of emitted dilatons from a single loop

$$N \sim \begin{cases} (4\pi)^{-1}\Gamma\Gamma_\phi \alpha^2\, G^2\, \mu^3 t^2, & \epsilon \gg 1, \\ (4\pi)^{-1}\Gamma\Gamma_\phi \alpha^2\, G^2\, \mu^3 t^2 \epsilon^2, & \epsilon \ll 1. \end{cases} \tag{10}$$

It is convenient to introduce further the relative abundance of dilatons, $Y_\phi = n_\phi(t)/s(t)$, where $n_\phi(t)$ is the density of dilatons and $s(t)$ the entropy density. In the radiation epoch, the entropy density is given by

$$s(t) = 0.0725[\mathcal{N}(t)]^{1/4} \left(\frac{M_{\rm Pl}}{t}\right)^{3/2}, \tag{11}$$

where $\mathcal{N}(t)$ is the effective number of spin degrees of freedom and $M_{\rm Pl}$ the Planck mass. Loops which decay at time t contribute to Y_ϕ the following abundance of dilatons,

$$Y_\phi(t) \sim \frac{n\,N}{s} \sim \begin{cases} p^{-1}\,\epsilon^{1/2}\,\zeta\,\Gamma_\phi \alpha^2\,(G\mu)^2\,(M_{\rm Pl}t)^{1/2}\mathcal{N}^{-1/4}, & \epsilon \gg 1, \\ p^{-1}\,\epsilon^{2}\,\zeta\,\Gamma_\phi \alpha^2\,(G\mu)^2\,(M_{\rm Pl}t)^{1/2}\mathcal{N}^{-1/4}, & \epsilon \ll 1. \end{cases} \tag{12}$$

The expression (12) differ from the one obtained in Ref. [5] by a factor p^{-1} for $\epsilon \gg 1$ and by a factor $p^{-1}\,\epsilon^{3/2}$ for $\epsilon \ll 1$. Equation (12) is valid as long as the loop sizes are smaller than the critical size $l_c = 4\pi/m_\phi$, or $t_c \lesssim 4\pi/(\Gamma\, G\mu\, m_\phi)$ for $\epsilon \gg 1$ and $t_c \sim 4\pi/(\epsilon\,\Gamma\, G\mu\, m_\phi)$ for $\epsilon \ll 1$. Substituting these times into Eq. (12) we find

$$Y_\phi(t) \sim \begin{cases} 20\,p^{-1}\,\epsilon^{1/2}\,\alpha^2\,(G\mu)^{3/2}\,(M_{\rm Pl}/m_\phi)^{1/2}, & \epsilon > 1, \\ 20\,p^{-1}\,\epsilon^{3/2}\,\alpha^2\,(G\mu)^{3/2}\,(M_{\rm Pl}/m_\phi)^{1/2} & \epsilon < 1. \end{cases} \tag{13}$$

In the last expression we replaced also the strong inequalities $\epsilon \gg 1$ and $\epsilon \ll 1$ by $\epsilon > 1$ and $\epsilon < 1$ to include the range $\epsilon \sim 1$.

The derivation of the expression (13) has been made for the scaling regime of the network. This assumes in particular that plasma friction is negligible. Since for times $t_* \lesssim 1/M_{\rm Pl}\,(G\mu)^2$ the motion of cosmic strings is heavily dumped by

the surrounding plasma, Eq. (13) is only valid for

$$G\mu > \begin{cases} \Gamma\, m_\phi/(4\pi\, M_{\rm Pl}), & \epsilon > 1, \\ \epsilon\, \Gamma\, m_\phi/(4\pi\, M_{\rm Pl}), & \epsilon < 1. \end{cases} \tag{14}$$

3 Observational bounds and the detection of gravitational waves

Having derived the abundance of dilatons (13) we can now apply different astrophysical constraints on the parameters of cosmic strings. First, ultra-light dilatons are excluded by tests of the gravitational inverse square law [10],

$$m_\phi > 10^{-3}\mathrm{eV}\,. \tag{15}$$

Another limit comes from the bounds on the density of dark matter. Dilatons interact with the gauge bosons through their mass terms in the Lagrangian, $\mathcal{L} = (1/2)\alpha_F\, \kappa\phi\, F^2_{\mu\nu}$, where $\kappa = \sqrt{8\pi}/M_{\rm PL}$ and α_F parameterizes deviations of the coupling at low-energies from the tree-level coupling at the string scale. The exact value of α_F is model-dependent, but generally of order one or larger [11]. Therefore dilatons decays into gauge bosons with the lifetime [5]

$$\tau_\phi \sim 3.3\times 10^{13}\mathrm{s}\left(\frac{12}{N_F}\right)\left(\frac{\mathrm{GeV}}{m_\phi}\right)^3\,, \tag{16}$$

where N_F is the number of gauge bosons with masses below $m_\phi/2$ and we assumed $\alpha_F = 1$.

If the lifetime of dilatons is larger than the age of the universe, $\tau > t_0 \sim 4\times 10^{17}$ s, the total density of dilatons is bounded by the observed density of dark matter. According to the WMAP observations, the relative fraction of dark matter is $\Omega_m h^2 = 0.13$ [12], where $h = 0.7$ parameterizes the Hubble constant. Thus

$$\Omega_\phi h^2 = \frac{n_\phi\, m_\phi\, h^2}{\rho_{\rm cr}} < 0.13, \tag{17}$$

where $\rho_{\rm cr}$ is the critical density of the universe. Using (17) one can easily obtain

$$Y_\phi < \frac{0.13\,\rho_{\rm cr}}{h^2\, s(t_0)\, m_\phi} \simeq 4.5\times 10^{-10}\left(\frac{\mathrm{GeV}}{m_\phi}\right). \tag{18}$$

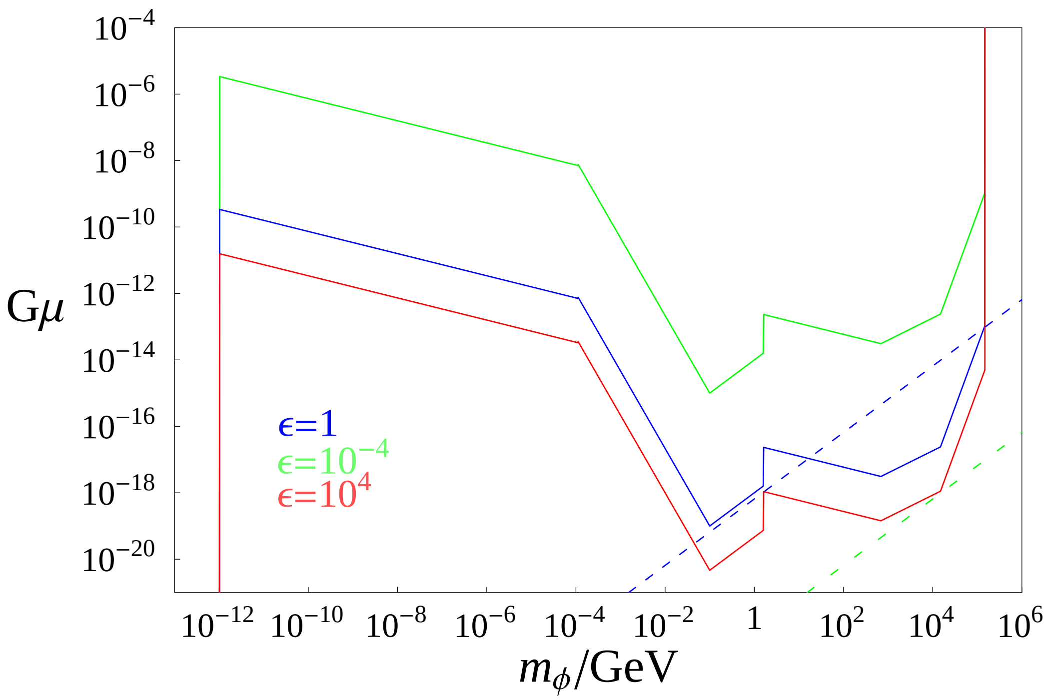

Fig. 1. A log-log plot of upper bounds on $G\mu$ as function of m_ϕ for several values of ϵ; the other parameters were chosen to have their "standard" values: $\alpha = 1$ and $p = 1$. Below the dashed lines, plasma friction invalidates our approximation.

This constrain is valid for $\tau > t_0$ or dilaton masses $m_\phi < 0.1$ GeV. For lifetimes $\tau_\phi < t_0$ one can use observations of the diffuse γ-ray background [13]. Decaying dilatons produce photons with number density $n_\gamma \sim n_\phi(t_0)\, t_0/\tau_\phi$. Using this relation and the approximate value for the upper bound of total energy density of photons with energy > 1 MeV, $\rho_\gamma \sim 2 \times 10^{-6}$ eV/cm^{-3} [13], we find

$$Y_\phi < \frac{\tau_\phi\, \rho_\gamma}{t_0\, s(t_0)\, m_\phi} \sim 7 \times 10^{-22} \left(\frac{\mathrm{GeV}}{m_\phi}\right)^4 . \tag{19}$$

For lifetimes $t_{\rm dec} < \tau < t_0$ of the dilaton (which corresponds to dilaton masses $0.1\,\mathrm{GeV} < m_\phi < 1.6$ GeV), where $t_{\rm dec} \sim 10^{13}$ sec is the decoupling time, we can also use the constraints from the density of γ-ray background. The energy density of photons created due to decay of dilatons at a time τ is $\rho_\gamma(\tau) = m_\phi n_\phi(\tau) = m_\phi\, Y_\phi\, s(\tau)$. Noting that the energy density of radiation scales with the time as $\rho_\gamma \propto t^{8/3}$, we obtain

$$Y_\phi < 3 \times 10^{-16} \left(\frac{m_\phi}{\mathrm{GeV}}\right) . \tag{20}$$

For the lifetime of dilaton $\tau_\phi < t_{\rm dec}$ we obtain constraints on Y_ϕ from the abundances of ^{4}He, ^{3}He, D and ^{6}Li. Using the results of Ref. [14] we can smoothly interpolate the results for decaying of relic particles and approximate

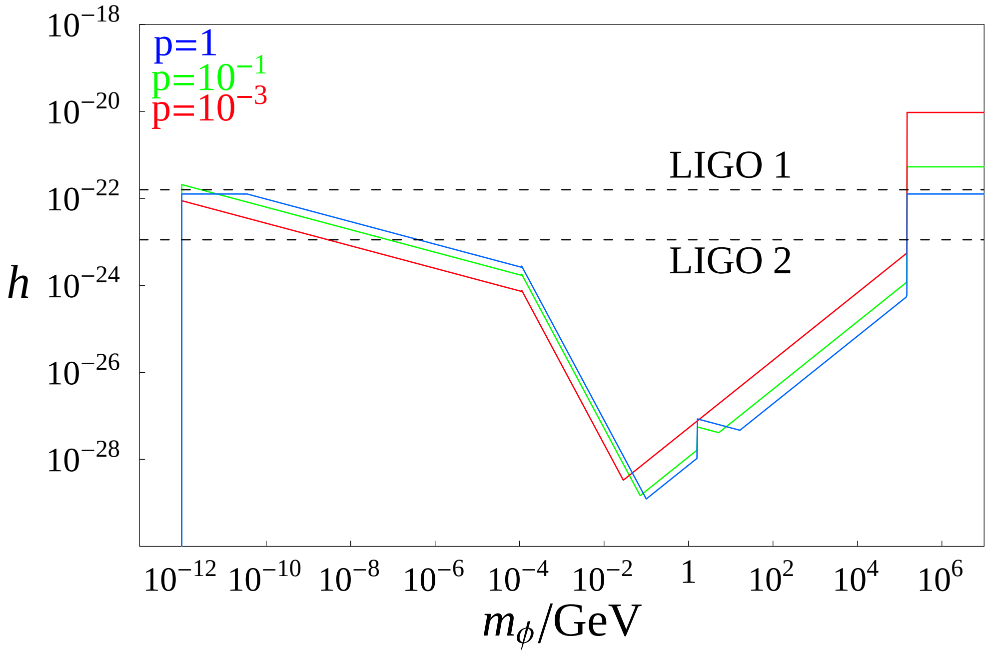

Fig. 2. Maximal allowed values for the amplitude h of the gravitational wave signal emitted in the LIGO frequency band, $f \sim 150$ Hz, versus the dilaton mass m_ϕ. The parameters of cosmic superstring network are chosen to be $\epsilon = 1$ and $p = 1$ (blue curve), $p = 10^{-1}$ (green curve), $p = 10^{-3}$ (red curve). The sensitivity of LIGO is shown in the upper dashed line (initial configuration) and lower dashed line (advanced configuration).

the maximal allowed abundance in the following way:

$$Y_\phi < \begin{cases} 10^{-14} m_\phi^{-1}, & 1.6\,\mathrm{GeV} < m_\phi < 690\,\mathrm{GeV}, \\ 5.5 \times 10^{-19}\, m_\phi^{1/2} & 690\,\mathrm{GeV} < m_\phi < 15\,\mathrm{TeV}, \\ 9 \times 10^{-38} m_\phi^5 & 15\,\mathrm{TeV} < m_\phi < 150\,\mathrm{TeV}, \end{cases} \tag{21}$$

For very heavy dilaton masses, $m_\phi > 150$ TeV, there are no limits on the abundance of dilatons.

Substituting the resulting expression for the dilaton abundance (13) into the obtained constraints for Y_ϕ (18–21), one can find the constraints on the string tension μ as function of dilaton mass m_ϕ, coupling constant α, the probability p of reconnection of long strings, and the parameter ϵ, determining the size of the closed loops produced by the network. For the "standard" values $p = 1$ and $\epsilon = 1$ the resulting Fig. 1 is similar to the one in Ref. [5]. The constraints on $G\mu$ are slightly more stringent, because of more precise observational data. However, the bounds on $G\mu$ may be strongly modified in comparison with Ref. [5] by the multiplier $p^{-1}\,\epsilon^{1/2}$ for $\epsilon > 1$ and $p^{-1}\,\epsilon^{3/2}$ for $\epsilon < 1$.

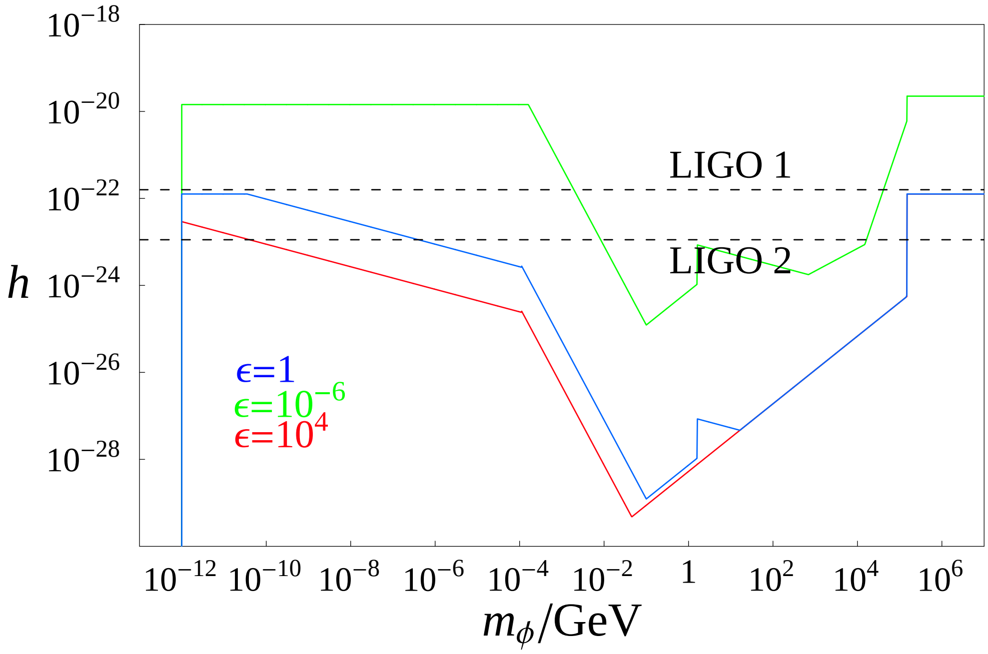

Fig. 3. Maximal allowed values for the amplitude h of the gravitational wave signal emitted in the LIGO frequency band, $f \sim 150$ Hz, versus the dilaton mass m_ϕ. The parameters of the cosmic superstring network are chosen to be $p = 1$ and $\epsilon = 1$ (blue curve), $\epsilon = 10^{-6}$ (green curve), $\epsilon = 10^4$ (red curve). The sensitivity of LIGO is shown in the upper dashed line (initial configuration) and lower dashed line (advanced configuration).

Next we apply the obtained constraints on the values of $G\mu$ to examine the prospects to detect gravitational wave bursts (GWB) from the cosmic superstring network. For each value of the dilaton mass m_ϕ, we find the maximal allowed value $G\mu$ and then, using the results of [8], the maximal possible signal for GWBs. We choose as frequency of the wave signal 150 Hz, i.e. the frequency range preferable for LIGO and assume one GWB event per year. The results for the maximum possible amplitude of the GWB signals are shown in Fig. 2 for different values of parameter p and in Fig. 3 for different values of ϵ. The sensitivity of the gravitational wave interferometer LIGO (and its advanced configuration) is also shown in these figures. We can see that the constraints from dilaton radiation significantly restrict the prospects for the discovery of cosmic strings by the detection of GWBs. In particular, for the range of parameters $\epsilon = 1$ and $10^{-3} < p \leq 1$, GWBs from a string network would hardly be detected by LIGO for dilaton masses 10^{-6} GeV $< m_\phi < 10^5 GeV$. Only in the case that the typical size of closed string loops is much smaller than in the standard scenario ($\epsilon \ll 1$), the prospects to detect GWBs from cosmic super strings improve: For instance, in the case of $p = 1$ and $\epsilon = 10^{-6}$ the detection of GRBs is possible for dilaton masses 10^{-12}GeV $\lesssim m_\phi \lesssim 10^{-2}$ GeV and for $m_\phi < 10^4$ GeV. In the case of very small values of ϵ, $\epsilon < 10^{-10}$, there are no limits on the GWBs amplitude coming from the dilaton abundance. We also checked the limits on GWBs coming from the stochastic gravitational

wave background [15]. However, this bounds is modified only by a factor < 3 in the range of dilaton masses $m_\phi < 150$ TeV and cosmic string parameters, $10^{-3} < p \leq 1$ and $10^4 < \epsilon < 10^{-6}$, therefore the Figs. 2 and 3 change only slightly.

4 Conclusions

We have examined the emission of dilatons by a network of cosmic superstrings. Depending on the particular mechanism of supersymmetry breaking, one expects a dilaton mass between 10 eV and 10 TeV if supersymmetry solves the hierarchy problem. We have derived stringent limits on the string tension as function of the dilation mass, cf. Fig. 1, for the case of a non-standard evolution of a string network. We have found that for dilaton mass in the favoured range between 10 eV and 10 TeV and values of the string tension $10^{-12} \lesssim G\mu \lesssim 10^{-7}$ predicted in Ref. [7], the detection of a gravitational wave signal from cosmic superstrings is only possible when the evolution of the string network deviates strongly from the standard case.

Acknowledgments

We are grateful to Alex Vilenkin for useful comments. This work was supported by the Deutsche Forschungsgemeinschaft within the Emmy Noether program and the Russian Foundation for Basic Research, grant 04-02-16757-a.

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